Math Supplement #2 Additional Challenging Problems

1. Whenever *x* is a positive number such that $9^{3x-1} = \left(\sqrt{3}\right)^{x}$, what is the value of *x* ?
   A. 2/11
   B. 4/11
   C. 11/4
   D. 11/2
   E. 11

2. Whenever *x* is a positive integer such that $9^{2x-3} = \left(\sqrt{3}\right)^{5x}$, what is the value of *x* ?
   A. 2
   B. 3
   C. 4
   D. 5
   E. 6

3. Whenever *x* is a positive integer such that $8^{2x-4} = \left(\sqrt{2}\right)^{9x}$, what is the value of *x* ?
   A. 4
   B. 6
   C. 8
   D. 12
   E. 16

4. Whenever *x* is a positive integer such that $16^{x-3} = \left(\sqrt{2}\right)^{7x}$, what is the value of *x* ?
   A. 6
   B. 8
   C. 12
   D. 24
   E. 36

5. Whenever *j* and *k* are positive integers such that $\left(\sqrt{2}\right)^{j} = 16^{k}$, what is the value of $\frac{j}{k}$?
   A. 1/8
   B. 1/4
   C. 2
   D. 4
   E. 8

6. Whenever *j* and *k* are positive integers such that $\left(\sqrt{5}\right)^j = 125^k$, what is the value of $\frac{k}{j}$?
   A. 1/6
   B. 1/2
   C. 2
   D. 4
   E. 6

7. Whenever *j* and *k* are positive integers such that $\left(\sqrt{3}\right)^j = 81^k$, what is the value of $\frac{k}{j}$?
   A. 1/8
   B. 1/4
   C. 2
   D. 4
   E. 8

8. If *x* and *a* are positive rational numbers such that $x^{3a} = 6$, then $x^{6a} =$?
   A. 3
   B. 6
   C. 9
   D. 18
   E. 36

9. If *x* and *a* are positive rational numbers such that $x^{2a} = 2$, then $x^{6a} =$?
   A. 2
   B. 4
   C. 6
   D. 8
   E. 16

10. What is the amplitude of the function $f(x) = 4\cos(3x + \pi)$?
    A. 4
    B. π
    C. 3
    D. 4/3
    E. ¾

11. Given that $(x + 3)$ and $(x - 2)$ are factors of the quadratic expression below, what are the values of *a* and *b* ?

$$x^2 + (a + 3)x + a + b$$

A. $a = -1 \; and \; b = -2$
B. $a = -1 \; and \; b = -5$
C. $a = -2 \; and \; b = -4$
D. $a = -4 \; and \; b = 2$
E. $a = -4 \; and \; b = -2$

12. Given that $(x + 4)$ and $(x - 1)$ are factors of the quadratic expression below, what are the values of *a* and *b* ?

$$x^2 + (a + 1)x + a + b$$

A. $a = -4 \; and \; b = 0$
B. $a = 2 \; and \; b = -4$
C. $a = 2 \; and \; b = -6$
D. $a = 3 \; and \; b = -4$
E. $a = 3 \; and \; b = -6$

13. The expression $\sin^2 \Theta - 3 + \cos^2 \Theta$ is equivalent to:
    A. -2
    B. 2
    C. 3
    D. 4
    E. 5

14. Assuming all values are defined, the expression $csc^2 \, x * tan \, x * cos \, x$ is equivalent to which of the following?

    (Note: $csc \, x = \frac{1}{\sin x}$)

    A. 1
    B. $\sec x$
    C. $\csc x$
    D. $\sin x$
    E. $\tan x$

15. Assuming all values are defined, the expression $\frac{(cot^2 x * sin x)}{\cos x}$ is equivalent to which of the following?

(Note: $cot\ x = \frac{1}{\tan x}$)

A. 1
B. $\sin^2 x$
C. $\csc x$
D. $\tan x$
E. $\cot x$

16. Assuming all values are defined, the expression $\sin x * \sec x * cot\ x$ is equivalent to which of the following?

(Note: $sec\ x = \frac{1}{\cos x}$ and $cot\ x = \frac{1}{\tan x}$)

A. 1
B. $\sec x$
C. $\csc x$
D. $\sin x$
E. $\tan x$

17. Assuming all values are defined, the expression $\sin x * \sec^2 x * cot\ x$ is equivalent to which of the following?

(Note: $sec\ x = \frac{1}{\cos x}$ and $cot\ x = \frac{1}{\tan x}$)

A. 1
B. $\sec x$
C. $\csc x$
D. $\sin x$
E. $\tan x$

18. Assuming all values are defined, the expression $\frac{\sin x * \sec x}{\tan x}$ is equivalent to which of the following?

(Note: $sec\ x = \frac{1}{\cos x}$)

A. 1
B. $\sec x$
C. $\csc x$
D. $\sin x$
E. $\tan x$

19. Assuming all values are defined, the expression $\frac{(1-\sin^2 x)*tan\ x}{\cos x}$ is equivalent to which of the following?

(Note: $cot\ x = \frac{1}{\tan x}$)

A. 1
B. $\sin x$
C. $\cos x$
D. $\csc x$
E. $\tan x$

20. Assuming all values are defined, the expression $\frac{\tan x}{\sin x * \sec x}$ is equivalent to which of the following?

(Note: $sec\ x = \frac{1}{\cos x}$)

A. 1
B. $\sec x$
C. $\csc x$
D. $\sin x$
E. $\tan x$

21. Assuming all values are defined, the expression $\frac{(1-\cos^2 x)*cot\ x}{\cos x}$ is equivalent to which of the following?

(Note: $cot\ x = \frac{1}{\tan x}$)

A. 1
B. $\sin x$
C. $\cos x$
D. $\csc x$
E. $\tan x$

22. Assuming all values are defined, the expression $\frac{(1-\cos^2 x)*cot\ x}{\sin x}$ is equivalent to which of the following?

(Note: $cot\ x = \frac{1}{\tan x}$)

A. 1
B. $\sin x$
C. $\cos x$
D. $\csc x$
E. $\tan x$

23. Assuming all values are defined, the expression $\frac{\sin x}{(1-\sin^2 x)*sec\ x}$ is equivalent to which of the following?

(Note: $sec\ x = \frac{1}{\cos x}$)

A. 1
B. $\sin x$
C. $\cos x$
D. $\sec x$
E. $\tan x$

24. Given $\sin\Theta = w$ where $0° \leq \Theta \leq 90°$, which of the following could be an expression for $\cos\Theta$?

A. $\frac{w}{\sqrt{1+w^2}}$

B. $\frac{1}{\sqrt{1-w^2}}$

C. $\frac{\sqrt{1-w^2}}{w}$

D. $\sqrt{1-w^2}$

E. $\frac{w}{\sqrt{1-w^2}}$

25. Given $\sin\Theta = w$ where $0° \leq \Theta \leq 90°$, which of the following could be an expression for $\cot\Theta$?

A. $\frac{w}{\sqrt{1+w^2}}$

B. $\frac{1}{\sqrt{1-w^2}}$

C. $\frac{\sqrt{1-w^2}}{w}$

D. $\sqrt{1-w^2}$

E. $\frac{w}{\sqrt{1-w^2}}$

26. Given $\cos\Theta = w$ where $0° \leq \Theta \leq 90°$, which of the following could be an expression for $\csc\Theta$?

A. $\frac{w}{\sqrt{1+w^2}}$

B. $\frac{1}{\sqrt{1-w^2}}$

C. $\frac{\sqrt{1-w^2}}{w}$

D. $\sqrt{1-w^2}$

E. $\frac{1}{w}$

27. Given $\cos\Theta = w$ where $0° \leq \Theta \leq 90°$, which of the following could be an expression for $\tan\Theta$?

A. $\frac{w}{\sqrt{1+w^2}}$

B. $\frac{1}{\sqrt{1-w^2}}$

C. $\frac{\sqrt{1-w^2}}{w}$

D. $\sqrt{1-w^2}$

E. $\frac{1}{w}$

28. Fourteen cars containing a total of 56 people crossed a toll bridge. Each of the 14 cars contained at least 1 person but no more than 6 people. At most how many cars contained exactly 3 people?
    A. 5
    B. 6
    C. 7
    D. 9
    E. 10

29. Twelve cars containing a total of 34 people crossed a toll bridge. Each of the 12 cars contained at least 1 person but no more than 4 people. At most how many cars contained exactly 2 people?
    A. 3
    B. 4
    C. 5
    D. 7
    E. 8

30. What is the value of *y* in the solution of the system of equations below?

$$x + 2y = a$$
$$x - y = b$$

A. $-\left(\frac{a+b}{3}\right)$
B. $a - b$
C. $b - a$
D. $\frac{a+b}{3}$
E. $\frac{a-b}{3}$

31. What is the value of *y* in the solution of the system of equations below?

$$x - 3y = a$$
$$x + y = -b$$

A. $-\left(\frac{a+b}{4}\right)$
B. $a - b$
C. $b - a$
D. $\frac{a-b}{4}$
E. $\frac{a+b}{4}$

32. The graph of $y = \sin x$ in the standard *(x,y)* plane is reflected over the *x*-axis, shifted down *a* units, and then shifted right π units. Which of the following equations represents the graph after the 3 transformations?
    A. $y = -a - \sin(x - \pi)$
    B. $y = -a - \sin(x + \pi)$
    C. $y = a - \sin(x - \pi)$
    D. $y = \pi - \sin(x + a)$
    E. $y = \pi + \sin(x - a)$

33. The graph of $y = \sin x$ in the standard *(x,y)* plane is reflected over the *x*-axis, shifted down 2π units, and then shifted left *a* units. Which of the following equations represents the graph after the 3 transformations?
    A. $y = -a - \sin(x - 2\pi)$
    B. $y = -a - \sin(x + 2\pi)$
    C. $y = -2\pi - \sin(x - a)$
    D. $y = -2\pi - \sin(x + a)$
    E. $y = -2\pi + \sin(x - a)$

34. Find x-intercept of the line $y = -3x + 12$
    A. -4
    B. 3
    C. 4
    D. 12
    E. 15

35. Two numbers have a product of -18 and a sum of 0. What is the lesser of the two numbers?
    A. -3
    B. $-3\sqrt{3}$
    C. $-3\sqrt{2}$
    D. $-2\sqrt{3}$

36. Two numbers have a product of -27 and a sum of 0. What is the lesser of the two numbers?
    A. -3
    B. $-3\sqrt{3}$
    C. $-3\sqrt{2}$
    D. $-2\sqrt{3}$

37. Joe volunteers at a charity that collects donations and then uses the money to buy food for care packages. This week, he collected $130. Each care package will include canned meats and bags of flour in the ratio of 4:1. The cans cost $0.79 each and the bags of flour cost $2.99 each. Using the given ratio, what is the maximum number of complete meat/flour care packages Joe can make?
    A. 16
    B. 18
    C. 21
    D. 24
    E. 27

38. Saul volunteers at a charity that collects donations and then uses the money to buy food for care packages. This week, he collected $150. Each care package will include canned meats and bags of flour in the ratio of 2:5. The cans cost $0.59 each and the bags of flour cost $3.99 each. Using the given ratio, what is the maximum number of complete meat/flour care packages Saul can make?
    A. 4
    B. 5
    C. 6
    D. 7
    E. 8

39. A system of equations is given below. What is the value of *b* in the *(a,b)* solution to the system?

$$a = -3b - 4$$
$$a = b + 4$$

    A. -2
    B. 2
    C. 3
    D. 4
    E. 5

40. A system of equations is given below. What is the value of *b* in the *(a,b)* solution to the system?

$$a = 5b - 1$$
$$a = b + 3$$

    A. -1
    B. 1
    C. 2
    D. 3
    E. 4

41. Whenever *x* and *y* are nonzero, $\frac{16x^{10}y^4}{(8x^3y^7)(4x^2y^9)} =$?

A. $2x^2y^4$
B. $2x^5y^{12}$
C. $\frac{x^5}{2y^{12}}$
D. $\frac{x^2}{2y^4}$

42. Whenever *x* and *y* are nonzero, $\frac{(5x^3y^4)(4x^7y^8)}{10x^2y^3} =$?

A. $2x^5y^4$
B. $2x^8y^9$
C. $10x^5y^4$
D. $10x^8y^9$

43. Whenever *x* and *y* are nonzero, $\frac{(12x^3y^2)(6x^7y^4)}{4x^4y^8} =$?

A. $18x^2y$
B. $18x^6y^2$
C. $\frac{18x^6}{y^2}$
D. $\frac{18x^2}{y}$

44. Whenever *x* and *y* are nonzero, $\frac{(6x^8y^7)(7x^4y^9)}{14x^{24}y^8} =$?

A. $3x^2y^6$
B. $3x^{12}y^8$
C. $\frac{3y^6}{x^2}$
D. $\frac{3y^8}{x^{12}}$

45. In the standard (x, y) coordinate plane, the midpoint of the line segment with endpoints $(-2, -3)$ and $(4, 7)$ is:
    A. $(3, 4)$
    B. $(3, 5)$
    C. $(3, 2)$
    D. $(1, 5)$
    E. $(1, 2)$

46. In the standard (x, y) coordinate plane, the point $(2, 4)$ is the midpoint of the line segment with endpoints $(-4, -6)$ and:
    A. $(8, 10)$
    B. $(8, 14)$
    C. $(6, 10)$
    D. $(6, 14)$
    E. $(6, 16)$

47. Rebekah spends 35% of her monthly paycheck on her car, 45% on rent, 15% on food, and 5% on entertainment. If she were to make a circle graph of her spending, what will be the degree measure of the "food" sector?
    A. 18°
    B. 35°
    C. 54°
    D. 126°
    E. 162°

48. Joe spends 30% of his monthly paycheck on his car, 40% on rent, 20% on food, and 10% on entertainment. If he were to make a circle graph of his spending, what will be the degree measure of the "food" sector?
    A. 144°
    B. 108°
    C. 72°
    D. 36°
    E. 20°

49. Jane spends 35% of her monthly paycheck on her car, 45% on rent, 15% on food, and 5% on entertainment. If she were to make a circle graph of her spending, what will be the degree measure of the "car" sector?
    A. 18°
    B. 35°
    C. 54°
    D. 126°
    E. 162°

Answers:

1. B
2. C
3. C
4. D
5. E
6. A
7. A
8. E
9. D
10. A
11. C
12. C
13. A
14. C
15. E
16. A
17. B
18. A
19. B
20. A
21. B
22. C
23. E
24. D
25. C
26. B
27. C
28. D
29. D
30. E
31. A
32. A
33. D
34. C
35. C
36. B
37. C
38. D
39. A
40. B
41. C
42. B
43. C
44. D
45. E
46. B
47. C
48. C
49. D